Math 32A, Lecture 1
Multivariable Calculus

Sample Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books, notes, or calculators. Show all your work; partial credit will be given for progress toward correct solutions, but unsupported correct answers will not receive credit. Remember to make your drawings large and clear, and to label your axes.

Write your solutions in the space below the questions. If you need more space, use the back of the page. Do not turn in your scratch paper.

Name: ______________________________

UID: ______________________________

Section: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] Draw the parallelopiped spanned by $\mathbf{v} = \langle -3, 0, -3 \rangle$ and $\mathbf{w} = \langle 0, 1, -1 \rangle$, and $\mathbf{u} = \langle 1, 2, 2 \rangle$ in three-dimensional coordinates.

(b) [5pts.] Compute the volume of this parallelopiped **using only the geometric properties of, and not the algebraic formula for** the cross product. (You can take any dot products you feel appropriate.)

Problem 2.

(a) [5pts.] Which of the following are right-handed systems? Justify your answers.

1. $\{\mathbf{j}, \mathbf{k}, \mathbf{i}\}$
2. $\{\mathbf{k}, -\mathbf{i}, \mathbf{j}\}$
3. $\{\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\rangle, \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\rangle, \mathbf{k}\}$

(b) [5pts.] Demonstrate that the cross product is not associative; that is, give three specific vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ with the property that $\mathbf{v} \times (\mathbf{u} \times \mathbf{w}) \neq (\mathbf{v} \times \mathbf{u}) \times \mathbf{w}$. [Hint: You don't need to use very complicated vectors.]

Problem 3.

(a) [5pts.] Give an equation for the plane P that contains the lines $\mathbf{r_1}(t) = \langle 2, 1, 0\rangle + t\langle 1, 2, 3\rangle$ and $\mathbf{r}_2(s) = \langle 5, 2, 8\rangle + s\langle 3, 1, 8\rangle$.

(b) [5pts.] For each of the two lines below, decide whether the line is contained in P, intersects P in a single point, or does not intersect P. If you decide that the second option is correct, give the point of intersection.

$$\mathbf{r_3}(u) = \langle 1, 2, 0\rangle + u\langle 1, -3, 2\rangle$$
$$\mathbf{r_4}(v) = \langle 6, 5, -8\rangle + v\langle 2, 2, -4\rangle$$

Problem 4.

(a) [5pts.] Sketch the plane curves $\mathbf{c}(t) = \langle t^3 + 4t^2, t^2 - 1\rangle$ and $\mathbf{d}(s) = \langle s^2 - 1, s\rangle$. One of the intersection points between these curves is $(0, -1)$. Find the others.

(b) [5pts.] The *angle between two curves* is the angle between their tangent lines at the point of intersection. With this in mind, decide what the cosine of the angle between the two curves in part (a) is at each of the intersection points you found above.

Problem 5.

Consider the vector-valued function $\mathbf{r}(t) = \langle \cos(t), \cos(2t), \sin(t) \rangle$.

(a) [5pts.] Draw the projections of $\mathbf{r}(t)$ to the three coordinate planes, and use these to give a sketch of the space curve determined by $\mathbf{r}(t)$.

(b) [5pts.] Find the equation of the tangent line to $\mathbf{r}(t)$ at $t_0 = \pi$.